What's with the Gamma Match Equations?

Background
Recently I needed to attempt a gamma matching solution to an antenna, with a parasitic element, with coax feed and I wanted to find the sensitivity to the various parameters to make a selection of gamma dimensions. The current ARRL Antenna Book comes with a PC calculator application ("Gamma") that allows you to input the gamma wire sizes and spacing to then calculate the length and capacitance needed for those wires to provide a match to your specified feed line and unmatched antenna impedance. It turns out that there are gamma calculators supplied on CD with another ARRL book plus others have placed on the web for online use by helpful hams. (Sadly, the various calculators do not always give the same answers.) However, if you want to answer the question as to what the output impedance will be from a specified gamma match of known dimensions acting on a given input antenna impedance, that cannot be done directly with the calculators. Such a capability may be of interest for looking at the sensitivities to selection of gamma parameters and adjusting the length of the antenna driven element. Beyond that, inquiring minds just want to know.

Yet more sadly, the information on gamma matches in The ARRL Antenna Book (21st) and the ARRL ON4UN's Low-Band DXing book (4th) have some unclear drawings and inconsistent editing for designations of the parameters, which make the matter a bit confusing. There is also a question about the application for dipole-type antennas (including yagis) versus vertical-type antennas (including gamma-fed towers).

The Equations
So here is what you need to know to calculate gamma transform and matching on your own using the conventional model. First the diagram of the set up:

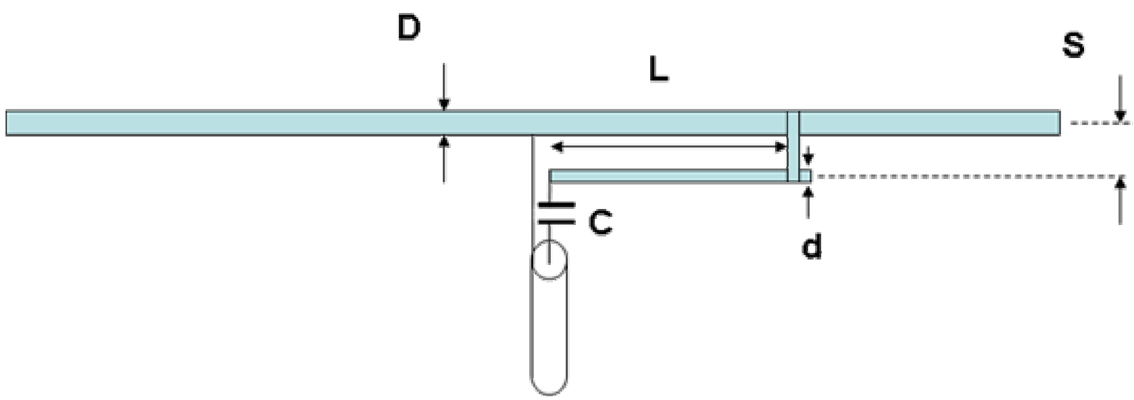

Figure 1. Gamma match schematic.

The basic quantities needed (consistent with common notation where possible) are:

Za - the complex impedance of the unmatched antenna (Za = Ra + j Xa, normally measured with dipole halves split)
S - <u>center-to-center</u> spacing of the circular antenna element to the circular gamma rod

D or d2 – diameter of the circular antenna element
d or d1 – diameter of the circular gamma rod
L – length of the gamma rod
C – the added series capacitance used to null any resulting inductive reactance

Not all authors are super careful in their drawings to indicate that the gamma rod spacing definition is center-to-center with the driven element, but this usage in the math seems to be universal.

The gamma rod, along with the driven element to which it runs parallel, can be viewed as a two wire transmission line with (potentially) different sizes of wire. This transmission line (as is well known among the EE types) has a characteristic impedance, almost always called Zo, and its value in ohms is

$$Zo = (376.73/2\pi)\ \mathbf{cosh^{-1}}(\ (4S^2-D^2-d^2)/(2Dd)\ ) \qquad (1)$$

Here the $376.73/2\pi$ is sometimes set to 60 (true value is 59.96…). The 376.73 ohms is the well known, nature-given, impedance of free space. This expression is fine for any consistent length units. The $\mathbf{cosh^{-1}}$ function is the inverse hyperbolic cosine function (aka, arc-cosh or acosh) that is not always available for a calculator or programming language. However, it can be evaluated exactly by

$$\mathbf{cosh^{-1}}(x) = \mathbf{log_e}(x+(x^2-1)^{1/2})\ . \qquad (2)$$

Now the conventional story is that a short length L of gamma match transmission line acts like a shunting inductance with reactive impedance of

$$j\ X_\Gamma = j\ Zo\ \mathbf{tan}\ (2\pi\ L / \lambda) \qquad (3)$$

where λ is the wavelength. Sometimes the quantity $2\pi L/\lambda$ is expressed in degrees of phase to describe the length. Of course, L and λ must be in the same units.

It turns out that tapping the driven element (no longer split) of the antenna off center with a gamma section of spacing S, driven element diameter D and gamma rod diameter d provides a "step-up" in impedance by a factor we will call "SU" here. It is far short of obvious that

$$SU = [1 + \mathbf{cosh^{-1}}(\ (4S^2-D^2+d^2)/(4Sd)\ )/\ \mathbf{cosh^{-1}}(\ (4S^2+D^2-d^2)/(4SD)\ )\ ]^2 \qquad (4)$$

This SU is the famous factor "Z Ratio," associated with a folded dipole, that is often plotted for various S/D (or S/d2) and D/d (or d2/d1) values such as below. Note that it is not dependent on the length of the gamma rod.

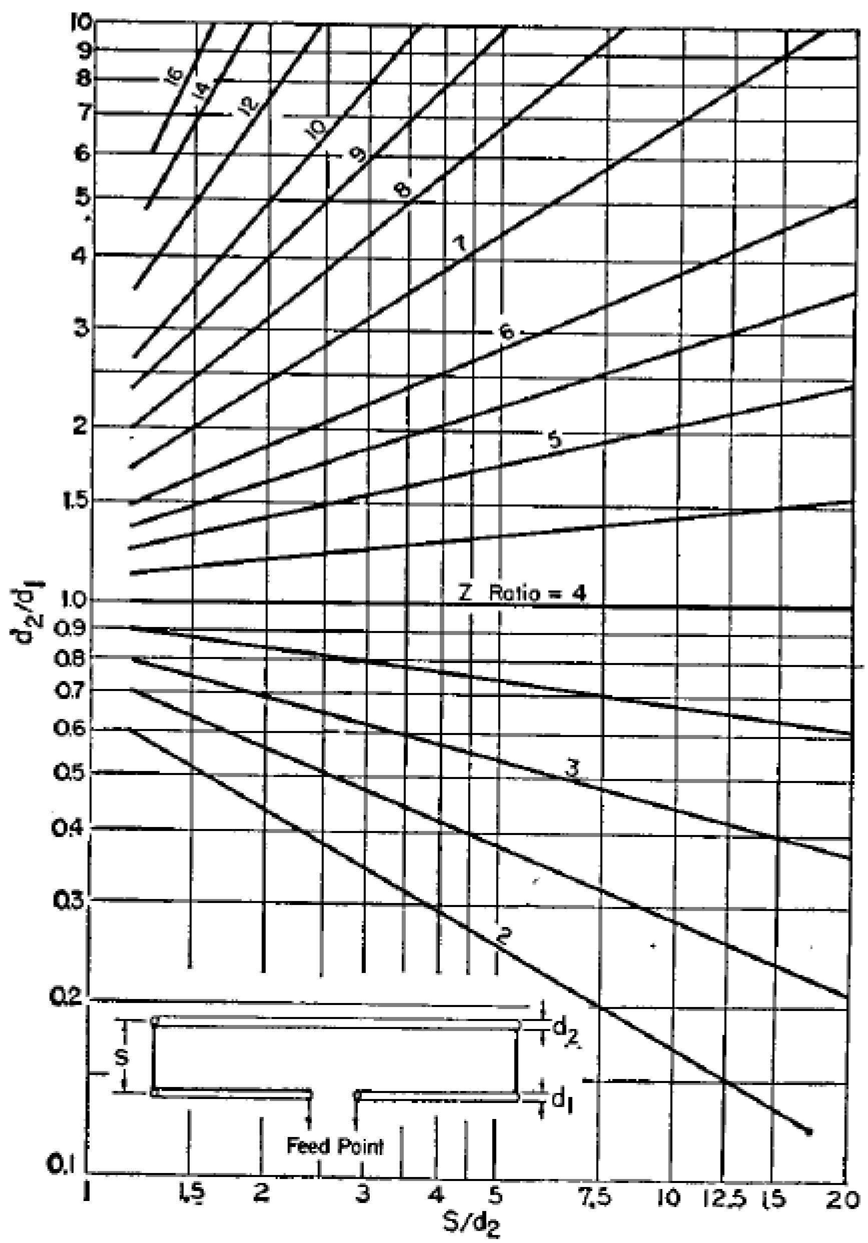

Figure 2. The Impedance Step-Up for folded dipole and other configurations.

This figure is taken from W3PG's article on gamma matching (QST, April 1969) - it is said to be originally taken from an unspecified edition of the ARRL Antenna Book. The results are consistent with the above equation for SU.

In ON4UN's Low-Band DXing book (4^{th} ed., p13-36) there is a reproduction of this plot, rather than a copy, BUT the vertical axis is incorrectly labeled d1/d2 where it should be d2/d1.

Now we have all the components needed to calculate the effects of a gamma match impedance transformation. The standard equivalent circuit used consists of a stepped up antenna impedance, Za*SU, shunted by the inductance impedance, j X_Γ , of the short transmission line formed by the added gamma rod as seen at the input end of the gamma rod. In later discussion there comes up the question as to whether the stepped up impedance should be Za*SU or Za*SU/2. This seems to be a point not fully resolved among gamma match publications. For the moment we will carry on without the ½ but it will be addressed later.

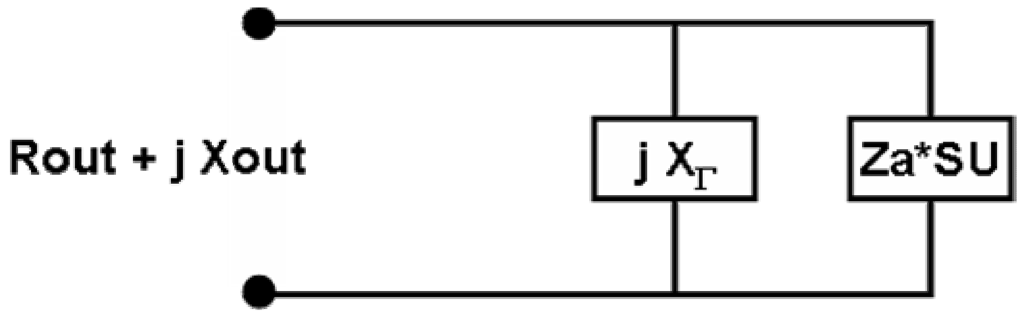

Figure 3. Conventional gamma match equivalent circuit.

So calling the complex output impedance Zout, and knowing how to combine parallel impedances, we arrive at the complex equation

$$1/\,Zout = 1/(Rout+jXout) = 1/(j\,X_\Gamma) + 1/(SU*(Ra+jXa)) \,. \qquad (5)$$

Thus given a gamma rod length to calculate X_Γ from (3) one can solve for the Zout with a simple bit of complex number arithmetic. So the matter of the transformation equations is resolved (with the possible exception of the ½.).

However, the frequent question is just how to select the gamma parameters to provide a good match to a feed line of known characteristic impedance (call it Ro). It is possible to do this by iteration (guess and correct) on the above equation but that is not necessary if you start with S, D, and d as specified quantities (which may still require some iteration if you don't like the answer you get). Generally there is not a unique solution for S, D, d, L and series capacitance C that you can find by just requiring Rout to be to equal to the feed line impedance and also requiring that Xout be nulled with that additional capacitor.

Solve for Length/Capacitance

If you specify desired S, D, and d (which then gives SU and Zo) and you know the unmatched antenna impedance, it is possible to solve directly for L and C (if a solution exists) by a direct algebraic approach with Eq (5). This can be done by inverting the expression to read

$$Rout+jXout = 1 / [\, 1/(j\,X_\Gamma) + 1/(SU*(Ra+jXa)) \,] \,. \qquad (6)$$

Then using just the REAL part, set the desired Rout to be the desired feed line impedance Ro (say 50 ohms), solve for X_Γ and finally convert that to L. This solving is straightforward but a tedious (as they say in math class) and it leads to a quadratic equation for which the solution with the correct sign must be taken, the one that provides a positive X_Γ. Then using this result you can return to the IMAG part of the equation and calculate the resulting Xout, which will be positive (inductive) if all is well. Finally using the frequency, f, and setting Xout = 1/(2πfC), you find the C that must be placed in series with Zout to cancel Xout and make it all look like a pure resistance to match the feed line with an SWR of 1:1. The equivalent circuit is below, where X_Γ is found from the calculated L needed to make Rout=Ro as indicated. The ARRL code Gamma (the Basic source code for it, GAMMA.BAS, is included) has the solution coded up in real number format. Beware that solutions are not always possible especially if Ra is small compared to Ro and/or if Xa is too inductive or even too close to being inductive. For a solution SU* $|Za|^2$/Ra>Ro is required, so smaller Za requires a bigger step up.

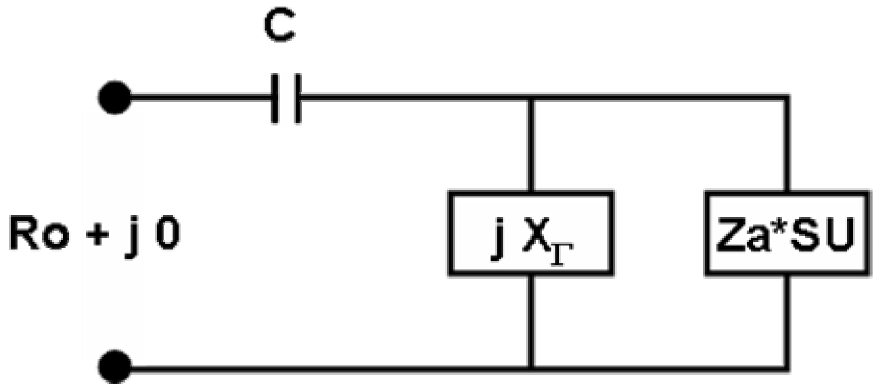

Figure 4. Fully gamma matched equivalent circuit.

The Calculators and Tables

There are two gamma calculators that have some degree of distribution plus another of interest that have been found. All three find L and C using S, D, d, feed line impedance and frequency as inputs. There is the ARRL Antenna Book CD with "Gamma," the ON4UN's ARRL Low-Band DXing book CD with "ON4UN's Low-Band DXing software" which has a GAMMA /OMEGA/HAIRPIN MATCHING feature (but no source code) and finally there is (or was) a MATLAB code found on the website of a McMaster University EE Professor. (After I sent an email inquiry about that code, which was not answered, public access to the MATLAB codes appears to have gone away. Hmmm.) Notes with the code indicates it is based on the book by Balanis.

Apart from two matters, the three codes appear to carry out essentially the same operations, although the details differ and there maybe some approximations in the ON4UN code that provide a modest deviation relative to the others. One matter is the factor of 1/2.

This factor is highlighted up front for the ON4UN code for which dipole-type antennas are the default use. For vertical antennas, the code gives the explicit instruction that this case, you should take the unmatched vertical antenna impedance, Za, and DOUBLE it when asked to type in the antenna impedance.

It turns out that if the ON4UN code is run in its vertical mode (instituted by the user doubling Za before input) it provides results that are quite similar to the (not doubled) ARRL "Gamma" code. Without that doubling they are rather different. The MATLAB code, which appears to carry out the solution steps described above, explicitly says it is working with a dipole. When this code is run, it generates results that are quite similar to the ON4UN code run in standard (dipole-like) mode. Finally if you go in and modify the MATLAB code (which transforms the unmatched antenna impedance to the tap point impedance by (Ra+jXa) -> SU*(Ra+jXa)/2 (note the factor of 1/2), by removal of the 1/2 in the transformation (or multiply the input Za by 2 before typing it in), the altered results are virtually identical to those from the ARRL "Gamma."

The second issue of interest is the discovery that the ARRL "Gamma" code will produce wildly incorrect results for some parameters. A brief investigation of the math suggests that the Gamma code is fine so long as SU*Ra > Ro is satisfied. When SU* $|Za|^2$/Ra> Ro > SU*Ra the full complex equations indicate that there are legitimate solutions but if an Ro in this range is input to Gamma, the results bear no relation to the actual solution and a negative capacitance is recommended (which may be a tipoff that there is a problem). This bad outcome is the result of the selection of the wrong quadratic equation root. Whatever the basis for the erroneous output, there are solutions that cannot be found with Gamma. Interestingly enough, if the ON4UN code is run in the double the Za mode, it can find the solutions for those Ro > SU*Ra. All this is discussed in grim detail in another note.

There are additional materials in the two ARRL books that need to be noted.

In the Antenna Book (21st ed) page 26-10, two examples of the "Gamma" code use are provided. The first example is for a yagi and the second for a shunt fed (and we assume grounded) tower which is a vertical. All the calculated values for the two examples quoted are completely consistent with the "Gamma" code from the CD going with that book. However, in the text for the yagi, it says "enter the choice for a dipole" but (the current?) "Gamma" does not offer any such option.

Finally, the ON4UN book (4th ed) contains Table 13-10 with yagi gamma match examples. Generally the results quoted are close to (but sometimes not identical to) the results from the ON4UN code that came with the same book. However, for the cases for Xa= -20 and 0, the quoted results for C are clearly wrong (and different from the ON4UN code calculated results). This same table was copied in the Silver article published in QST in Dec 2002. The ARRL has been informed of the issue of the table columns.

Dipole versus Vertical

There seems to be some undertone of technical disagreement on the matter of treatment of dipole-types and monopole-type verticals for gamma matching. The ON4UN code is quite explicit and in his book (see Fig 13-56, p13-36, lower right) the splitting of Za into equal parts is very clear.

Yet in the BASIC source code for the ARRL Gamma calculator there is a notation:

"12 REM Removed corrections RA/2 and XA/2 per W6NL, Apr 1, 2000"
which suggests that the different treatment was once considered and is now rejected.

I have exchanged some emails with Dave Leeson, W6NL, who has contributed a variety of antenna expertise in ARRL and other publications. He provided a preliminary copy of his in-progress technical article addressing some of the issues about gamma (and tee) matches. In the context of the current discussion, the upshot of W6NL's conclusions is that there should not be a factor of ½ appearing in the gamma transformation and the existence of it at all resulted from a misinterpretation of early work using monopoles that used a factor of 2 to convert to the dipole equivalent impedance. From W6NL's discussion, this seems to have resulted in a common but improper inclusion of the factor of ½, apparently taken by some from the Balanis book, for the transformed antenna tap point impedance (i.e., the apparently incorrect SU*Za/2) and this was then propagated into the gamma literature. Finally W6NL points out that for a monopole the factor ½ should also not be present when the Za used is the impedance of that monopole. (Again note that if the monopole impedance is taken as that for an equivalent dipole (i.e., twice the impedance of the monopole), then and only then should the ½ be used.)

Other's Experimental Data
I have not seen any reports of serious experimental data taken to verify the gamma transform theory, but then again this is a niche market. Examples seen in the ham domain generally have the property that a calculator is used to estimate L and C and then (sometimes much) adjustment ensues. Healey's article is a mix of theory and experiment but it appears that the theoretical approach has aspects different from conventional understanding, plus it is not really explained how the step-up factor is used, and then there are the dreaded Smith charts.

My motivation for looking at the gamma match arose from the matching needs of a two element parasitic vertical array constructed for 30 meters. This was encouraged by an article by Hulick (QST) who used a gamma match for his 80m array with shortened loaded wire elements. Unfortunately the author does not reveal what the starting unmatched antenna impedance actually is so it is hard to draw any firm conclusions there.

One pretty clean example of data was provided in W6NL's unpublished article where a half-folded dipole (i.e., one with only one of its elements was folded, and using the same tube size for the gamma rod as the driven element, d=D) was a part of a larger yagi system. Here is was found that the original (not folded) impedance (25 ohms) was transformed into a 4X impedance (100, not 50) by actual measurement. For this approximately quarter wave length gamma, the transmission line contribution drops out (X_Γ is very large) so this indicates that the factor of ½ should not be present.

"Calculated" Data
By way of qualification, the phrase "calculated data" is, of course, a oxymoron but they may provide insights. Generally, the models used are certainly not full solutions to Maxwell's equations and there are a number of approximations, omissions and fits based on the original modeler's judgment. With that in mind:

W6NL's unpublished article also mentions some instances where some antenna modeling colleagues have tested the much discussed factor of ½ by direct construction of gamma matches with wires as part of the models. The reports were always that the ½ factor was not appropriate.

EZNEC provides a ready tool to get a handle on some of the issues raised. Using it, a set of models were generated for a dipole, and variants, and a vertical, and variants.

Dipole
Test of gamma matching transformation using EZNEC with the following sequence of dipole configurations:

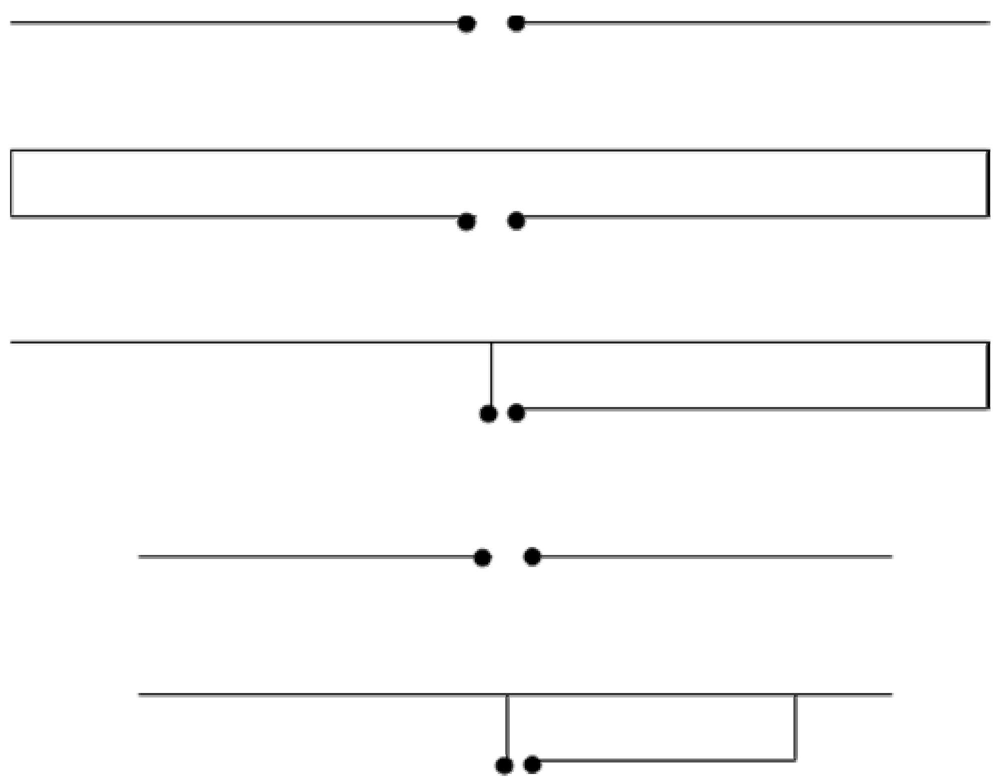

Figure 5. Dipole models and variants.

In EZNEC, first take a free space dipole, and for the sake of round numbers, make it 60' each leg with 1" diameter wires. Result is resonance at fo=3.95 MHz and Z(fo) = 72.1-j0.2 within the frequency resolution used. In all cases, the segments on the longer wires were approximately 2'.

Next, make it into a folded dipole with second wire also with 1" diameter and with separation of 2'. Now fo=3.85 MHz with Z(fo)= 286.4-j.07 (Note that 4 X 72 = 288 so this is very close to the expected result when SU=4 for this D=d case.) At 3.95 MHz, Z(3.95) = 309.4+j 87.

Now into less charted territory, we make the folded dipole into a HALF-FOLDED dipole so it is fed like a gamma match where the gamma rod extends all the way to the end of the full dipole. Now fo = 3.895 MHz and Z(fo) = 287-j0.3. And Z(3.95) = 297+j56. Therefore a half folded dipole (at least with SU=4 for D=d) will have the essentially the same impedance as a full folded dipole with that same 4X step up.

So this says that the gamma match transform over a full half dipole should provide a 4X impedance change. This goes directly to the old factor of 1/2 and it says that the 1/2 should not appear for this (dipole) case. (Note that for a dipole with quarter wave legs, the shunt inductance reactance X_Γ in the gamma transform becomes infinite and so it does not contribute.)

Next we can compare the way the impedance changes from a gamma transform versus EZNEC for some more general cases. Take the prior simple dipole and shorten it from 60' to 50' per leg. Now the Z(3.95) = 42.5 – j204.3 . This will now be an example of unmatched antenna impedance, Za, that can be changed with a gamma match.

Now we add a gamma feed to our shortened dipole in the standard way with a 2' separation and D=d=1". The gamma rod length, Lg, will be varied and the effects determined on the impedance at the output input of the center gamma rod feed. At the same time we can calculate (see Eq (6)) the expected resultant impedance using the same numerical gamma transform discussed before which is the same as appears in the gamma match calculators. This will be done using two versions of the transformation, one with the factor 1/2 included (such as the ON4UN dipole version) and then without that factor (such as the ARRL Gamma code). The tabulated results for the resistance and reactance at the gamma input for five gamma rod lengths are provided below.

Dipole	EZNEC		Xform / 2		Xform NO /2	
Lg (ft)	Rg	Xg	Rg	Xg	Rg	Xg
30	196	949	2007	-272	190	860
25	67	588	811	999	75	522
20	27.4	387	184	585	32.6	364
15	11.3	255	50.5	317	13.5	235
10	4.25	158	13.4	165	4.7	139

Table 1. Shortened dipole gamma transform effects.

To ease the analysis, these results are now plotted with the two transformed versions (with and without the ½) normalized with the EZNEC values for both Rg and Xg. Of course, a result that is close to EZNEC will have a value near 1 (the green line). Red is for factor 1 and blue is for factor ½.

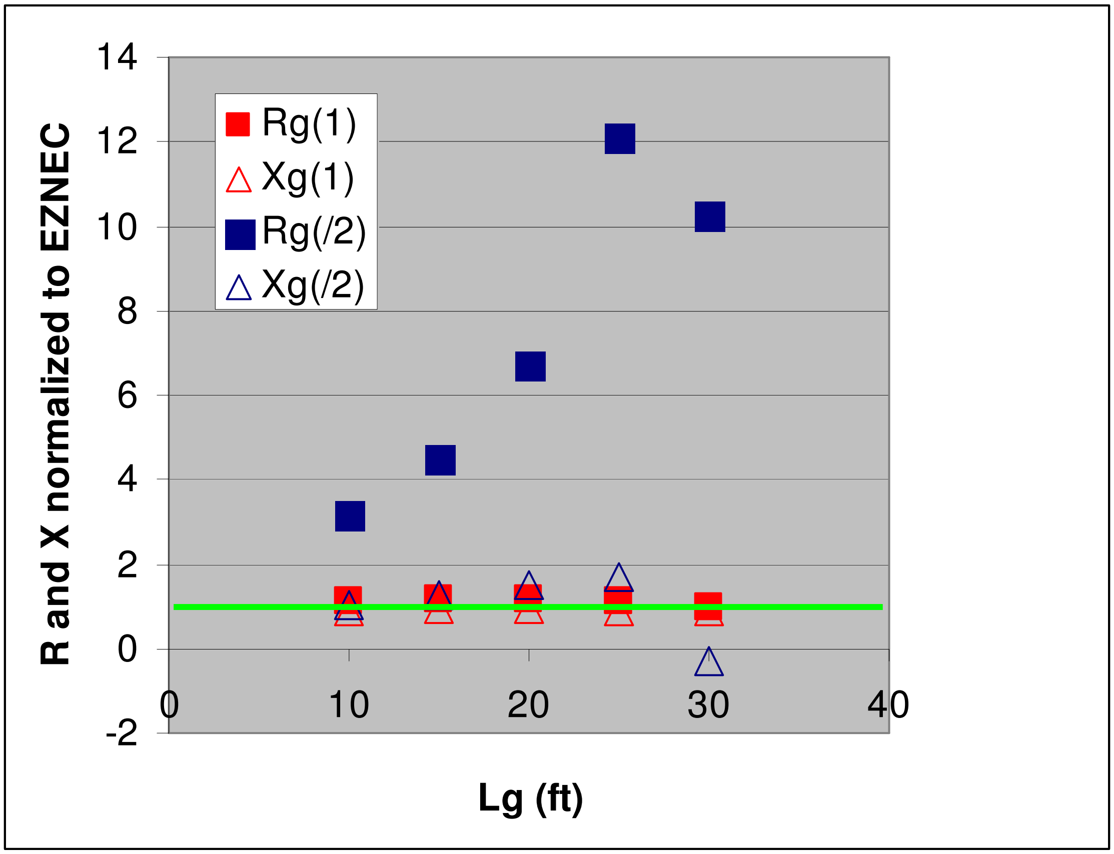

Figure 6. Shortened dipole model results for gamma transforms with EZNEC versus gamma length variations.

It seems clear that the transform <u>without</u> the ½ agrees very well with EZNEC while the case with ½ included is very poor. Note that EZNEC does not have any implicit model for a gamma match so this simulation is done purely by the addition of wires to the model. (If wire loops are too small, EZNEC will refuse to do the computation but the loops used here are acceptable to the code – how accurate the results may be is subject to discussion.)

Vertical

Next we test the gamma matching transformation using EZNEC with the following sequence of perfect ground vertical configurations:

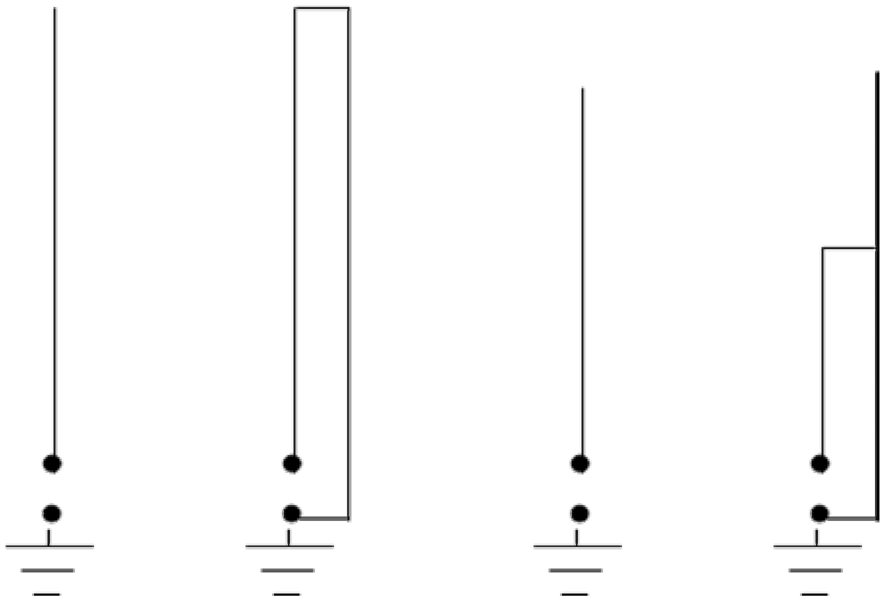

Figure 7. Vertical models and variants.

Start with the simple vertical at 60'. Here EZNEC gives fo=3.95 MHz and Z(fo) = 34.5 – j.07 for the frequency resolution used.

Using the same S= 2' and D=d= 1" from the dipole case, the folded vertical of the same 60' height gives fo=3.85 and Z(fo) = 137+ j.02 . (At 3.95, Z = 147.8 + j42.) Immediately we see the X4 in Z occurring from the folding suggesting there is no ½ factor for the vertical case either.

To be sure this is not some odd special case, we again shorten the pure vertical to 50' to give a complex Z and apply the gamma configuration. For this shortened version, EZNEC gives for 3.95 MHz, Z = 20.4- j 97.9 . This will be the unmatched impedance input, Za, for the transforms.

Again a sequence of gamma rod lengths is applied in EZNEC and the resulting impedance, Zg, at the bottom end of the gamma rod is found. The results are in the table below along with the results of doing the numerical calculation of the nominally equivalent gamma transform with and then without the factor of 1/2.

Vertical	EZNEC		Xform / 2		Xform NO /2	
Lg (ft)	Rg	Xg	Rg	Xg	Rg	Xg
25	537	1175	212	-404	990	980
20	99.1	563	504	-490	214	612
15	28.6	307	774	400	56	326
10	9.04	168	78.1	265	14.5	168
5	1.86	75.7	6.91	82	2.41	69

Table 2. Shortened vertical gamma transform effects.

A plot of the values again normalized to EZNEC is provided below. Red is for factor 1 and blue is for factor ½.

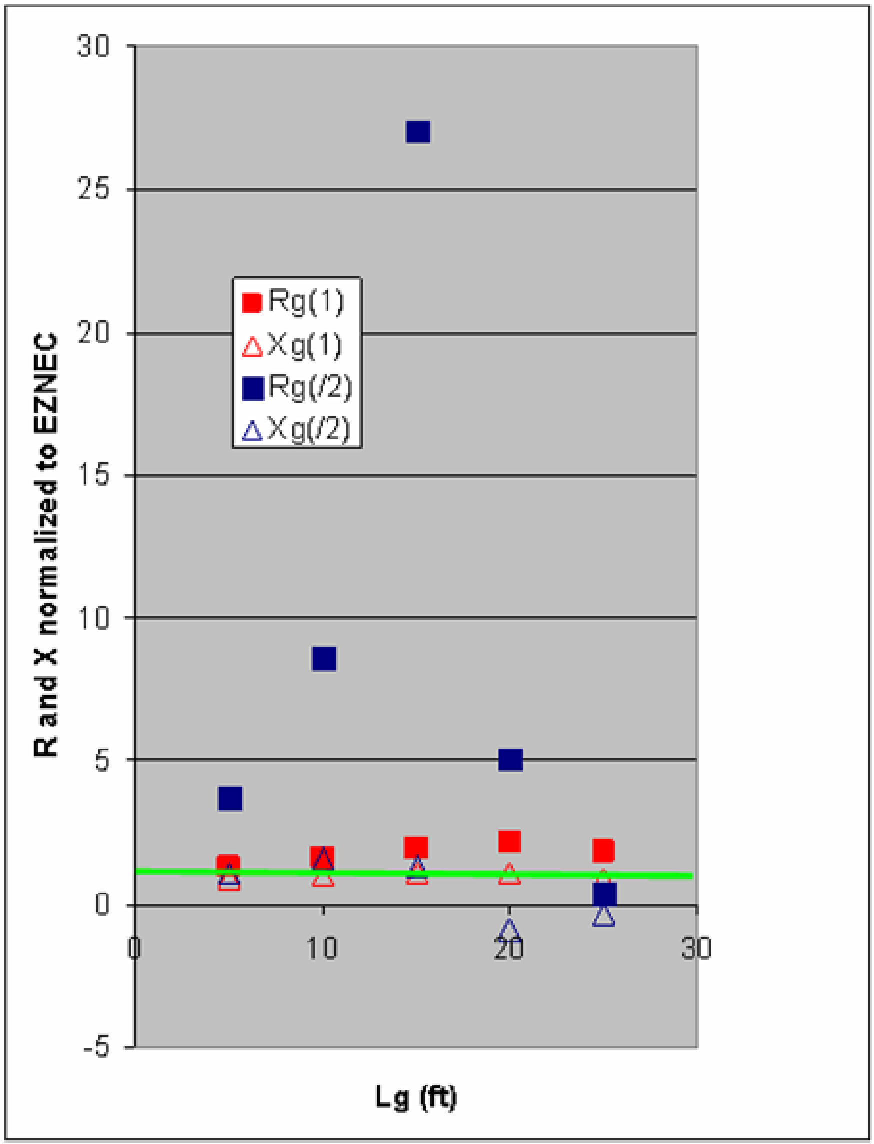

Figure 8. Shortened vertical model results for gamma transforms with EZNEC versus gamma length variations.

For this case the transform without the 1/2 again is clearly in much better agreement (but certainly not perfect) with EZNEC than that with the 1/2, but the agreement is not as good as for the dipole. (W6NL points out that the effective radius of the element is altered by adding the gamma match and this impacts the impedance somewhat and these effects that are not included.)

Again EZNEC is a model rather than a full up solution of Maxwell's equations and it has a variety of approximations. Furthermore, the gamma transform equations are from an “equivalent” circuit that is probably not really quite equivalent. Perfect agreement is not expected.

Still all this provides a pretty compelling argument that the gamma match transform equations should never have a factor of ½ to divide the unmatched antenna impedance either for a dipole or for a vertical (again - unless you happen to specify the vertical impedance as the equivalent dipole impedance, by doubling).

Low Grade Experimental Data

The following results were found when an initial simple effort to determine the needed capacitance in the subject gamma match failed. I ran a gamma calculator using nominal S, D, d values to estimate L and C. The unit was constructed without the C (to be purchased once determined) and the impedance was measured with no C in place (i.e., Xc=0). The plan was to verify the resistive part (Rout, planned to be 50) and find the residual inductive impedance - then add the standard series C that would cancel the inductive part. BUT, it was found that the Rout was huge, not at all close to 50. My initial reaction was that somehow having a DC short instead of C was the culprit. That was tested by using a large capacitor instead – same result.

From this, one effort at a gamma match on my two element parasitic 30m vertical array used nominal dimensions from materials at hand (S=8.25”, D= .75” and d=.375”) with L at 35”. For the example provided here, the frequency is 10.0 MHz and the unmatched Za for the driven element was 18-j23.[1] The gamma calculators (ARRL Gamma or ON4UN in vertical mode) indicate for matching to 50 ohms, L=33” and Xc=100 ohms are needed, while L=35” would match a 58 ohm feedline using Xc=101 ohms (157 pF). [Here, and in the following, Xc is always given as a positive number but the properly signed capacitive reactive is really –Xc, of course.]

Measurements were done with fixed value capacitors to generate a fairly complete range of Rout/Xout values for their variation with the series capacitor impedance Xc. One example is shown in the following plots for the gamma rod length, L, of 35”. These data were taken under non-professional poorly controlled conditions (including use of MFJ-259B) that might be representative of typical amateur conditions. Red is for measured data and blue is for the equivalent of the GAMMA code calculation with the standard gamma match equivalent circuit model.

[1] When the two element vertical array was deployed for actual use, the matching technique used was the “hairpin” method, which was quite effective. The unmatched impedance was manipulated but adjusting the length of the driven element to allow a good hairpin (really a coil) match.

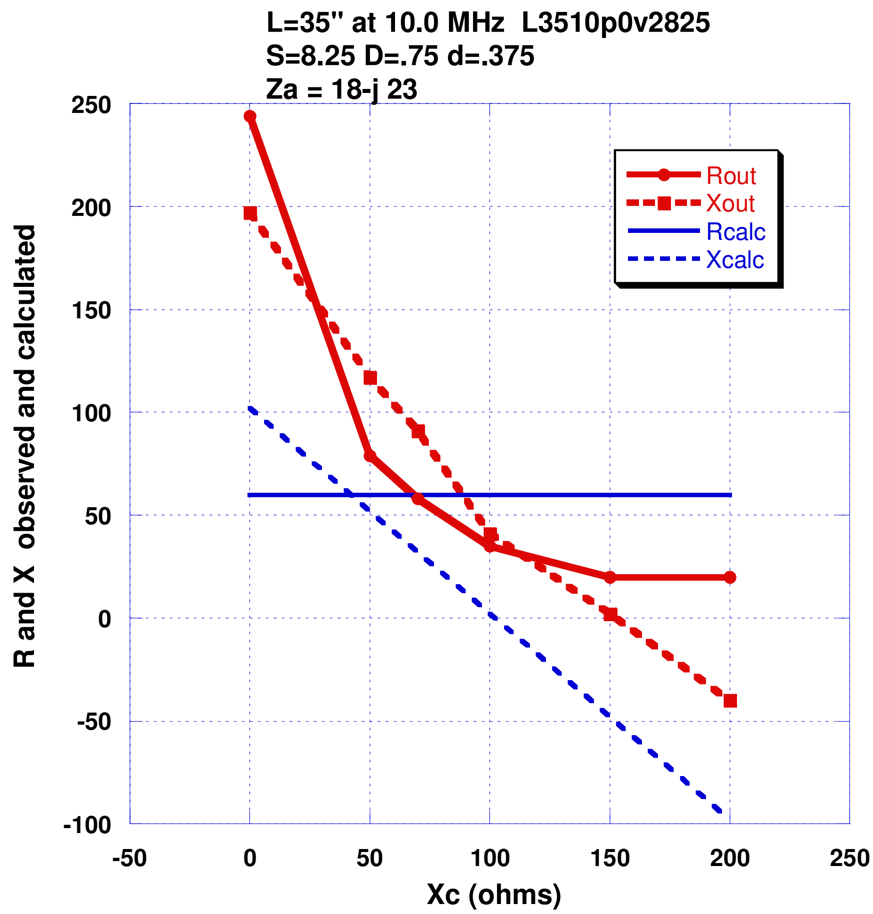

Figure 9. Experimental effects (red) of varying the series capacitor for a gamma network compared to the gamma equivalent circuit expectation (blue).

Note that the calculated R is independent of Xc, as expected, while the measured value is strongly dependent on Xc, especially for small Xc. It is comforting that the X curves (observed and calculated) have nearly the same slope (value ~1) as expected but the two are offset by about 50 ohms. The calculated result can be regarded as the required gamma match found by standard formulas for Z=18-j23 to a fictional 58 ohm cable where they would indicate the need for L=35" with an Xc of 101 ohms. However, the measured gamma transformed antenna, for Xc~100, instead of yielding the calculated Z=58+j0 was actually Z=35+j41. (The resonant value, Xout=0, is found to be about R=20 ohms so these gamma match parameters would match a 20 ohm feedline.) Either these measurements are way off, or the standard gamma matching equivalent circuit is not very good (or both).

The same effect of significant variation of Rout with changes in Xc was seen over several other sloppy experimental examples not reproduced here. This may be the aspect of gamma matching that creates such frustration for the user. Still the experimental arrangement for these results were substantially lacking in quality control so additional work was done on a cleaner case as reported next.

Better Quality Experimental Data – Short Dipole

The previously discussed experimental data were taken under rather ad hoc conditions with limited controls. I have attempted to have a cleaner case with better controls by using a simple shortened (about 5% short) horizontal dipole made from 6' lengths of telescoped 3/4" and 5/8" tubing, and then finding the gamma match transform effects on it to make a more convincing test.

First the frequency was selected to have an available 50' coaxial cable (RG8U) feedline length be a multiple of an electrical half wavelength. The MFJ-259B indicated that this cable is 3/2 electrical wavelength at 22.10 MHz. Use of this frequency then avoids impedance transformation by the feedline so the impedance at the antenna feedpoint is nearly the same as the impedance that is measured at the transmitter end of the coax. 22.1 MHz is a fairly handy frequency with a practical dipole length and then raising it to the available 20' removes much of the ground effect on antenna impedance.

First the full dipole was modeled with EZNEC which, for the tubing used, showed a free space resonance (X=0) at 22.1 for a length of 10.73 feet for each half. This was assembled and initially fed directly with the coax with the two halves split but with a cable coil for a common mode choke. The intent was to use this as verification of the modeling and as a starting point for length selection.

In order to find the resonance, it was necessary to take the MFJ measured values over a frequency range at the end the cable and transform those impedances (using TLW and tweaking the length of one of the RG8 choices to be consistent with the MFJ data showing 22.10 to be the 3/2 λ point) to those at the antenna. These data taken with the antenna away from the house (by manual mast balancing and pricey XYL data taking assistance) showed the resonance at ~ 22.6 MHz at 20' height. The EZNEC model at 20' height with real ground gives a resonance of 22.3. When the mast was mounted at the edge of the house (and about 15' from a multiband vertical, broadside) the resonance as at 22.3 MHz. This seemed reasonable so the dipole half length was reduced to 10.19 feet for a target resonance (with the house mount) of 23.5 MHz at 20' height with a target impedance of 70-j30 at 22.1 MHz.

The shortened dipole was raised at 20' and the impedance at 22.1 was found to be 76-j26 (close enough). The sign of X (which the MFJ withholds) was determined by inserting an additional short length of coax and comparing the change to that expected using TLW. This then is the raw mismatched antenna (after the two halves are joined) that will be used as the basis to test gamma matches and transforms.

The inner region of the dipole is 3/4" tubing and a length of 3/8" tubing was available as the gamma rod. The ARRL GAMMA code was run and it says to match to 50 ohms (a fully arbitrary choice here) using a gamma center-to-center spacing 8", a length of 27.6" and a capacitor of 53pF are needed. The ON4UN code (no doubling) says a length of 18.7" and a capacitor of 83pF are needed to match to a 45.5 ohm feedline. (It would have been a bit cleaner if this has been 50 but an unfortunate minor oversight occurred - still the conclusions are not affected. The ON4UN code gives 19.6” and 80pF for the correct impedance for the 50 ohm match. However 18.7 was used in the experiments and the results are fully valid, albeit for an unconventional, notional 45.5 ohm cable.) The ON4UN code says that for L=27.6”, under these conditions, the transformed output will be 91+j0 for C=69pF (104 ohms), determined by iteration, as compared to GAMMA which says 50+j0 for C=53pF (136 ohms).

Based on this, two cases were investigated, one with 27.6” and the other with 18.7” gamma rod length. A variable air capacitor was used to provide the capacitance (with many ups and downs of the antenna for the changes). For each case the output impedances of the gamma matches were measured at 22.1 MHz at the end of the calibrated 50’ cable with the antenna at 20’ height over a range of capacitances. (The capacitor was “calibrated” using the MFJ instrument and it pretty much tracked the expected values based on the claimed range of the capacitor (12-313pF) and the extent to which it was meshed.)

Note again that by working at 22.1 MHz, the feedline impedance does not affect the measured impedance and we can interpret the results to indicate what feedline impedance (of any length) would provide a match to the antenna. Figure 10 provides the results for the 27.6” gamma length.

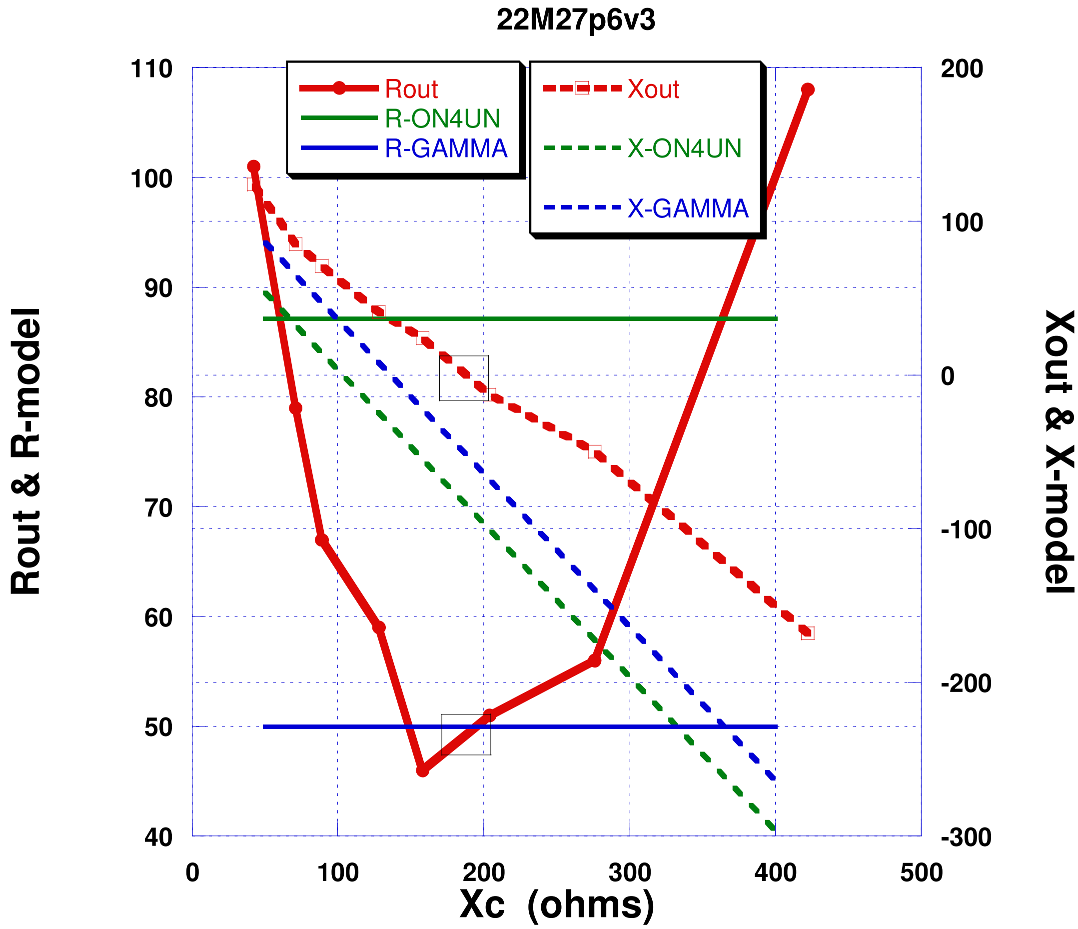

Figure 10. Experimental effects (red) of varying the series capacitor for a gamma network with L = 27.6" compared to the standard gamma equivalent circuit expectation (in blue) at 22.1 MHz. The ON4UN code equivalent (with the ½ factor) prediction is shown in green.

The experimental Xout=0 point is for Xc of about 185 ohms (C=39 pF). At this value of Xc, the Rout is about 48 ohms. The boxes indicate the X and R locations for this match. The standard equivalent circuit (GAMMA in blue) indicates a C of 53 pF to reach a Zout of 50 ohms where the data show Zout of about 48 for a C of 39 pF. This is not too different.

The ON4UN equivalent circuit predicts an Rout of 88 ohms (just a bit different from the ON4UN code which says 91) with a C of 69pF (104 ohms) to null the inductance. This R is quite different from the experimental data.

Figure 11 provides the corresponding experimental results for the 18.7" gamma length compared with the GAMMA model equivalent circuit and the ON4UN model equivalent circuit.

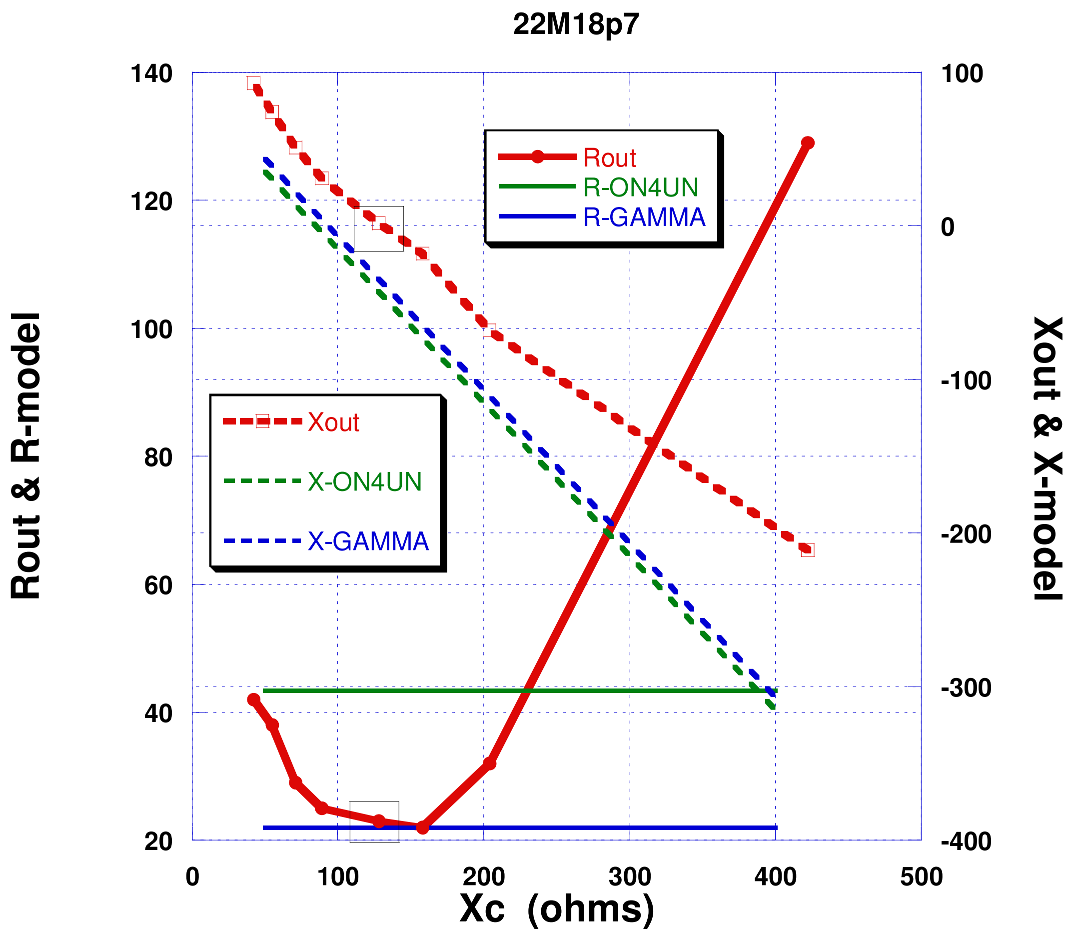

Figure 11. Experimental effects of varying the series capacitor for a gamma network with L = 18.7" compared to the two gamma equivalent circuit calculations at 22.1 MHz (GAMMA blue and ON4UN green).

At Xout=0 the data show Rout of about 24 ohms again indicated by the boxes. The GAMMA code gives R of 23 ohms while the ON4UN code predicts 43 ohms (just a bit different from the result expected from the standard equivalent circuit with the Zin divided by 2 – suggesting again that the ON4UN code is not quite the same as the GAMMA code with a factor of 1/2 added, but very similar).

Clearly the GAMMA prediction is quite similar to the experimental result aside from a difference of about 30 ohms in the capacitive reactance. The ON4UN calculations are again off by about a factor of 2 for the (vital) resistive component.

One aspect of the data which is very striking is that Rout is not at all flat with variation in the value of the series capacitor. This is in sharp contrast to the equivalent circuit model of the matching network. However, in the region around Xout near zero, the variation of R tends to show a broad minimum so locally (once you find it) the equivalent circuit may be okay

Less striking is there is some offset in the observed versus calculated reactance of the network. Generally the reactance behaves much like expected for a series capacitor, with a constant slope, aside from the offset which may represent some stray reactance contributions from the less than pristine experimental conditions, or another aspect of the limit of the standard equivalent circuit..

Bottom Line

1. If you take Figure 1 to define the parts of a gamma match, you can use Equations 1, 3, 4, and 5 to determine the effects of adding a gamma match to an antenna ("The Equations") using the standard gamma match equivalent circuit.

2. At least some available gamma match calculators do not correctly evaluate the standard gamma match equivalent circuit. The post 2000 version of the ARRL "Gamma" appears to be okay except for cases where the desired feedline impedance exceeds SU*Ra. The ON4UN calculator supplied with Low-Band DXing appears to be incorrect when used for dipole-like antennas. It is okay if the input unmatched antenna impedance is doubled (as recommended only for verticals) for all antennas. The tables presented in ON4UN's book for gamma matching yagis appear to be incorrect due to the factor of ½ issue. Furthermore the Xa = -20 and 0 ohms columns for the Ra = 25 ohm example clearly do not agree with ON4UN's calculator. These same questionable table data were copied into Silver's Dec 2002 QST article.

3. It seems quite possible that the standard gamma match transform equivalent circuit is not a very good model for real world antennas and this may be the reason that, in addition to the factor of ½ issue, hams who have used the gamma match have often faced a lot of cut and try before making it work, if ever.

4. From the more careful testing described before, at least for dipole-like antennas, the ARRL Gamma code leads to a fairly good starting point for the gamma match parameters that might be used, but used with a variable capacitor for tuning, to get a decent match without any tweaking of the gamma rod length or the driven element length. Those who have an emotional need for a 1:1 SWR will need to struggle with multiple trips up the tower.

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